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RELIABILITY ACCEPTANCE SAMPLING PLANS BASED UPON PRIOR DISTRIBUTIONS

SEP 76 A L GOEL, A M JOGLEKAR

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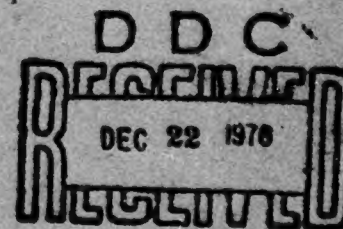
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Final Technical Report  
September 1976



**RELIABILITY ACCEPTANCE SAMPLING PLANS BASED UPON PRIOR DISTRIBUTION  
Design of Testing Plans**

**Syracuse University**

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Determination of the Prior Distribution," provides the means for determining the parameters of the prior distribution from existing data, and discusses the reason for using an inverted gamma. Volume IV, "Design of Testing Plans," provides instructions for establishing a test time and number of allowable failures based on the prior distribution and the selected risks. Volume V, "Sensitivity Analyses," shows the effects on the test parameters caused by changes in the prior parameters.

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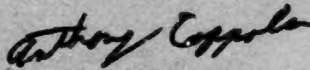
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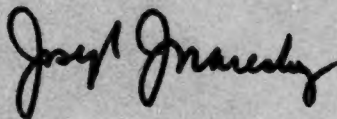
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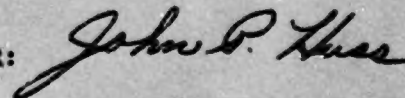
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## PREFACE

This report is the fourth of a set of five presenting the results of part of the work done under contract number F-30602-71-C-0312. The report is delivered to RADC in accordance with item A006 of the Contract Data Requirement List. Sponsorship and technical direction of this task originated in the Reliability and Maintainability Engineering Section (A. Coppola, Chief), Reliability Branch (D. Barber, Chief), within the Reliability and Compatibility Division (J. Naresky, Chief) of the Rome Air Development Center. Mr. Anthony Coppola was the Project Engineer who was technically supported by Mr. Jerome Klicn.

The titles of the reports on the subject "Reliability Acceptance Sampling Plans Based Upon Prior Distribution" are as follows:

- Volume I. Introduction and Problem Definition.
- Volume II. Risk Criteria and Their Interpretation.
- Volume III. Implications and Determination of the Prior Distribution.
- Volume IV. Design of Testing Plans.
- Volume V. Sensitivity Analyses.

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### ABSTRACT

Numerical and graphical procedures for the design of reliability acceptance sampling plans are developed in this report. The failure distribution is considered to be exponential with parameter  $\theta$  and the prior distribution of  $\theta$  is taken to be inverted gamma. These procedures are applicable when accept/reject decisions are to be made regarding a sequence of systems or lots. For specified combinations of producer-consumer risks, the numerical and the graphical procedures can be used for the design of a truncated single sample plan for a system. Other single sample plans can be obtained by using the equivalence relations derived in this report. The use and advantages of the graphical procedure are illustrated via numerical examples.

#### ACKNOWLEDGMENTS

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## 1. INTRODUCTION

This report presents numerical and graphical procedures for the design of reliability acceptance sampling plans for the case when the failure distribution is exponential, i.e.

$$f(t|\theta) = \theta^{-1} \exp(-t/\theta); t \geq 0, \theta > 0$$

where  $t$  denotes the time of failure and  $\theta$  is the mean time between failures (MTBF). The parameter  $\theta$  is considered to be a random variable with known prior frequency distribution  $g(\theta)$ . This prior distribution is assumed to be the conjugate inverted gamma density

$$g(\theta) = \gamma^\lambda \theta^{-(\lambda+1)} \exp(-\gamma/\theta) / \Gamma(\lambda); \gamma, \lambda, \theta > 0.$$

The successive lots or systems are taken to have different (constant but unknown)  $\theta^i$ ,  $i = 1, 2, 3, \dots$ ; drawn independently from  $g(\theta)$ .

These procedures are applicable when accept/reject decisions are to be made regarding a sequence of systems or a sequence of lots. It is assumed that the decisions are based on data subject to random fluctuations, that a definite measure of loss is associated with each of the two decisions when they are inappropriately taken and that each lot or system is sentenced on its own merit without regard to the decisions taken about other lots or systems.

Following four types of single sample plans are considered:

- (i) truncated plan with replacement
- (ii) truncated plan without replacement
- (iii) censored plan with replacement
- and (iv) censored plan without replacement.

For a lot, which consists of a large number of components, all the four plans are feasible. For a system, which can be repaired as new upon failure,

100% inspection is assumed and plans without replacement are infeasible. For specified combinations of producer-consumer risks, the numerical and the graphical procedures can be used for the design of a truncated single sample plan for a system. Other single sample plans can be obtained by using the equivalence relations derived in this report.

The use and advantages of the graphical procedure are illustrated via four examples. Specifically, the examples illustrate the design of plans and the exploration of the design region, use of multiple risk criteria, design with engineering constraints and design with partial information.

## 2. EQUIVALENCE RELATIONS

Plans with identical  $P(A|\theta)$  vs  $\theta$  curves are identical i.e. plans with the same operating characteristic (O.C.) are equivalent. We now determine the conditions under which the different types of single sample plans have the same operating characteristic curve as the truncated plan for a system. That is, we derive the equivalence relationships between a single sample truncated plan and the other types of plans.

### 2.1 Plans with Replacement

#### 2.1.1. Truncated Single Sample Plan For A System

Under this scheme a single system is placed on a life test of duration  $T$ . Without loss of generality, repair is assumed to be instantaneous. If the observed number of failures,  $r$ , is less than or equal to a value  $r^*$  i.e.  $r \leq r^*$ , the system is accepted. Otherwise, it is rejected. Since the failure distribution is exponential, the number of failures in fixed time  $T$  has a Poisson distribution. Then, the probability of acceptance for a given  $\theta$ , can be written as:

$$P(A|\theta) = \sum_{r=0}^{r^*} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \quad (1)$$

### 2.1.2. Truncated Plan For A Lot

In this scheme a sample of  $n$  items from a lot is placed on a life test of duration  $t_0$ . Failed items are immediately replaced. If  $r \leq r'$ , the lot is accepted, otherwise it is rejected.

The failure process can be considered as a Poisson process with rate  $(n/\theta)$ . Hence,

$$P(A|\theta, n) = P(r \leq r' | \theta, n) = \sum_{r=0}^{r'} \frac{e^{-nt_0/\theta} (nt_0/\theta)^r}{r!} \quad (2)$$

It is easy to see that the O.C. curves given by Equations (1) and (2) are identical if  $nt_0 = T$  and  $r' = r^*$ . Thus, the desired plan for this situation will be given by  $(t_0 = T/n, \text{ and } r' = r^*)$ . A suitable value of  $n$  is chosen to make the sample representative of the lot and/or from cost considerations.

### 2.1.3 Censored Plan For A System

A system is life tested until  $r'$  failures occur. If the total test time  $t' > T'$ , the system is accepted; otherwise it is rejected. If  $t_1, t_2, \dots, t_{r'}$  represent the times to failure then  $t' = \sum_{i=1}^{r'} t_i$  has a gamma distribution given by,

$$f(t' | \theta) = \frac{e^{-t'/\theta} t'^{(r'-1)}}{\theta^{r'} \Gamma(r')} .$$

Hence,

$$P(A|\theta) = P(t' > T'|\theta) = \int_{T'}^{\infty} \frac{e^{-\frac{t'}{\theta}} (t')^{r'-1}}{\theta^{r'} \Gamma(r')} dt'$$

If  $t'' = t'/\theta$ , then

$$P(A|\theta) = \int_{T'/\theta}^{\infty} \frac{e^{-t''} t''^{r'-1}}{\Gamma(r')} dt'' = \sum_{r=0}^{r'-1} \frac{e^{-T'/\theta} (T'/\theta)^r}{r!} \quad (3)$$

Equations (3) and (1) are identical if  $r' = r^* + 1$  and  $T' = T$ . Hence the desired plan is  $(T, r^* + 1)$ .

#### 2.1.4 Censored Plan For A Lot

A sample of  $n$  items from a lot is life tested until  $r' \leq n$  failures occur. If the total test time  $t' > T'$ , the lot is accepted; otherwise it is rejected.

It is shown in Epstein (1960) that  $\frac{2t'}{\theta} \sim \chi_{2r'}^2$

Hence,

$$P(A|\theta) = P(t' > T'|\theta) = P\left(\frac{2t'}{\theta} > \frac{2T'}{\theta}\right)$$

$$= \int_{\frac{2T'}{\theta}}^{\infty} \frac{1}{\Gamma(r')} \frac{1}{2^{r'}} x^{r'-1} e^{-x/2} dx,$$

where

$$x = \frac{2t'}{\theta}.$$

If  $t'' = \frac{x}{2}$ , then

$$P(A|\theta) = \int_0^{\frac{T'}{\theta}} \frac{e^{-t''} t''^{r'-1}}{\Gamma(r')} dt'' = \sum_{r=0}^{r'-1} \frac{e^{-T'/\theta} (T'/\theta)^r}{r!} \quad (4)$$

Comparing Equations (1) and (4), the required plan is obtained by taking  $r' = r^* + 1$  and  $T' = T$ .

## 2.2 Plans Without Replacement

### 2.2.1 Truncated Plan For A Lot

In this scheme a sample of  $n$  items from a lot is life tested for duration  $t_0$ . Failed items are not replaced. If  $r \leq r'$ , the lot is accepted; otherwise it is rejected.  $r$  has a binomial distribution and

$$P(A|\theta) = P(r \leq r'|\theta) = \sum_{r=0}^{r'} \binom{n}{r} (1 - e^{-t_0/\theta})^r (e^{-t_0/\theta})^{n-r} \quad (5)$$

Epstein (1954) shows that for small  $\alpha, \beta$  and  $\theta_0/t_0 \geq 3$ , approximately

$r' = r^*$  and

$$t_0 = - \frac{\theta_0 \chi^2_{1-\alpha}(2r^*)}{2r^*} \ln \left( \frac{n-r^*}{n} \right).$$

Since

$$T = \left\{ \frac{\theta_0^2 \chi^2_{1-\alpha}(2r^*)}{2} \right\}$$

we have  $t_0 = \frac{T}{r^*} \ln \left( \frac{n}{n-r^*} \right)$ .

### 2.2.2 Censored Plan For A Lot

$n$  items from a lot are life tested until  $r' \leq n$  failures occur. Failed items are not replaced. If the total test time  $t' \geq T'$ , the lot is accepted; otherwise it is rejected.

Epstein (1960) shows that, as in the replacement case,  $2t'/\theta \sim \chi^2_{2r'}$ . Hence the designed plan is  $r' = r^* + 1$  and  $T' = T$ .

The relationships between the above plans are summarized in Table 1.

TABLE 1

EQUIVALENCE RELATIONS

	SYSTEM		LOTS	
	<u>Truncated</u>	<u>Censored</u>	<u>Truncated</u>	<u>Censored</u>
With Replacement	$(T, r^*)$	$(T, r^* + 1)$	$(t_0 = T/n, r^*)$	$(T, r^* + 1)$
Without Replacement	-	-	$(t_0 = \frac{T}{r^*} \ln(\frac{n}{n-r^*}), r^*)$	$(T, r^* + 1)$

### 3. RISK CRITERIA

Since the parameter  $\theta$  is assumed to have a prior density  $g(\theta)$ , the producer's and the consumer's risks may be quantified in various ways. The definitions of the various risks are now given in terms of  $P(A|\theta)$  and  $g(\theta)$ . For the interrelationships and interpretations of various risks, the reader is referred to Goel and Joglekar (1976 b)

#### 3.1 Producer's Risks

##### Classical:

The probability of rejecting a system with  $\theta$  equal to the specified value  $\theta_0$  is equal to  $\alpha$ , i.e.

$$\alpha = P(R|\theta_0), \text{ or } 1 - \alpha = P(A|\theta_0) \quad (6)$$

##### Average:

The probability of rejecting a good system ( $\theta \geq \theta_0$ ) is equal to  $\bar{\alpha}$ , i.e.

$$\bar{\alpha} = P(R|\theta \geq \theta_0)$$

or

$$1 - \bar{\alpha} = \int_{\theta_0}^{\infty} \frac{P(A|\theta) g(\theta) d\theta}{\int_{\theta_0}^{\infty} g(\theta) d\theta} \quad (7)$$

Posterior:

The probability of a rejected system being good is equal to  $\alpha^*$ , i.e.

$$\alpha^* = P(\theta > \theta_0 | R)$$

or

$$\alpha^* = \frac{\int_0^{\infty} \{1 - P(A|\theta)\} g(\theta) d\theta}{1 - \int_0^{\infty} P(A|\theta) g(\theta) d\theta} \quad (8)$$

Probability of Rejection:

The probability that a randomly selected system will be rejected is prespecified. This probability represents the long range fraction of the submitted systems that will be rejected by the sampling plan and is given by

$$P(R) = 1 - \int_0^{\infty} P(A|\theta) g(\theta) d\theta \quad (9)$$

3.2 Consumer's Risks

Classical:

The probability of accepting a system with minimum acceptable MTBF  $\theta_1$  is equal to  $\beta$ , i.e.

$$\beta = P(A|\theta_1) \quad (10)$$

Average:

The probability of accepting a system of unacceptable reliability ( $\theta \leq \theta_1$ ) is equal to  $\bar{B}$ , i.e.

$$\bar{B} = P(A|\theta \leq \theta_1)$$

or

(11)

$$\bar{B} = \frac{\int_0^{\theta_1} P(A|\theta) g(\theta) d\theta}{\int_0^{\theta_1} g(\theta) d\theta}$$

Posterior:

The probability of an accepted system being bad ( $\theta \leq \theta_1$ ) is equal to  $\beta^*$ , i.e.

$$\beta^* = P(\theta \leq \theta_1 | A)$$

or

(12)

$$\beta^* = \frac{\int_0^{\theta_1} P(A|\theta) g(\theta) d\theta}{\int_0^{\theta_1} P(A|\theta) g(\theta) d\theta}$$

Alternate Posterior:

To provide a better control on the distribution of  $\theta$  in the accepted systems, we define an additional posterior risk,  $\beta^{**}$ , which specifies the probability that an accepted system will have an MTBF less than the specified value  $\theta_0$ . Thus,

$$\beta^{**} = P(\theta \leq \theta_0 | A)$$

or

$$\beta^{**} = \frac{\int_0^{\theta_0} P(A|\theta) g(\theta) d\theta}{\int_0^{\infty} P(A|\theta) g(\theta) d\theta} \quad (13)$$

The various risk criteria have been defined above in terms of  $P(A|\theta)$  and  $g(\theta)$ . Explicit expressions for the risks for all the four types of single sample plans can be obtained by substituting the corresponding expressions for  $P(A|\theta)$  derived earlier. Risk expressions for the truncated plan for a system are given below.

### 3.3 Expressions for Producer's Risks

Classical:

$$1 - \alpha = \sum_{r=0}^{r^*} \frac{e^{-T/\theta_0} (T/\theta_0)^r}{r!} \quad (14)$$

Average:

$$1 - \bar{\alpha} = \frac{\int_0^\infty \left\{ \sum_{r=0}^{r^*} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta}{\int_0^\infty \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta} \quad (15)$$

Posterior:

$$1 - \alpha^* = \frac{\int_0^\infty \left\{ 1 - \sum_{r=0}^{r^*} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta}{1 - \int_0^\infty \left\{ \sum_{r=0}^{r^*} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta} \quad (16)$$

Probability of Rejection:

$$P(R) = 1 - \int_0^\infty \left\{ \sum_{r=0}^{r^*} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta \quad (17)$$

### 3.4 Expressions for Consumer's Risks

Classical:

$$\beta = \sum_{r=0}^{r^*} \frac{e^{-T/\theta_1} (T/\theta_1)^r}{r!} \quad (18)$$

Average:

$$\bar{\beta} = \frac{\int_0^{\theta_1} \left\{ \sum_{r=0}^{\infty} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta}{\int_0^{\theta_1} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta} \quad (19)$$

Posterior:

$$\beta^* = \frac{\int_0^{\theta_1} \left\{ \sum_{r=0}^{\infty} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta}{\int_0^{\infty} \left\{ \sum_{r=0}^{\infty} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta} \quad (20)$$

Alternate Posterior:

$$\beta^{**} = \frac{\int_0^{\theta_1} \left\{ \sum_{r=0}^{\infty} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta}{\int_0^{\infty} \left\{ \sum_{r=0}^{\infty} \frac{e^{-T/\theta} (T/\theta)^r}{r!} \right\} \frac{\gamma^\lambda}{\Gamma(\lambda)} \theta^{-(\lambda+1)} e^{-\gamma/\theta} d\theta} \quad (21)$$

#### 4. NUMERICAL PROCEDURE FOR THE DESIGN OF PLANS

Plan design for a specified pair of producer-consumer risks is achieved by solving the corresponding pair of equations. Alternately, one may only consider the design of truncated plan for a system and obtain the remaining plans by using the equivalence relations given in Table 1.

As an example, if the design criteria are  $P(R)$  and  $\beta^*$  then given  $(\gamma, \lambda, \theta_1, \theta_0, P(R), \beta^*)$  Equations (17) and (20) can be solved to obtain  $T$  and  $r^*$ . Let  $T^* = T/\theta_0$ ,  $\gamma^* = \gamma/\theta_0$ ,  $K = \theta_0/\theta_1$  and  $\phi = \theta/\theta_0$ . Then Equations (17) and (20) become

$$P(R) = 1 - \int_0^\infty \left\{ \sum_{r=0}^{r^*} \frac{e^{-T^*/\phi} (T^*/\phi)^r}{r!} \right\} \frac{\gamma^{*\lambda}}{\Gamma(\lambda)} \phi^{-(\lambda+1)} e^{-\gamma^*/\phi} d\phi \quad (22)$$

and

$$\beta^* = \frac{\int_0^{1/K} \left\{ \sum_{r=0}^{r^*} \frac{e^{-T^*/\phi} (T^*/\phi)^r}{r!} \right\} \frac{\gamma^{*\lambda}}{\Gamma(\lambda)} \phi^{-(\lambda+1)} e^{-\gamma^*/\phi} d\phi}{\int_0^\infty \left\{ \sum_{r=0}^{r^*} \frac{e^{-T^*/\phi} (T^*/\phi)^r}{r!} \right\} \frac{\gamma^{*\lambda}}{\Gamma(\lambda)} \phi^{-(\lambda+1)} e^{-\gamma^*/\phi} d\phi} \quad (23)$$

Clearly, given  $(\gamma^*, \lambda, K, P(R), \beta^*)$  the values of  $T$  and  $r^*$  can be numerically obtained, if the solution exists.

The numerical procedure consists of searching for those values of  $T$  and  $r^*$  in the design region that satisfy the desired risk criteria. Let us call the consumer's risks  $\beta^*, \beta^{**}, \bar{\beta}$  as the left hand risks (LHR) and the producer's risks  $\alpha^*, \bar{\alpha}, P(R)$  as the right hand risks (RHR). It is intuitively clear that the LHR will increase with increasing  $r^*$  and de-

creasing  $T$ ; while the LHR will increase with decreasing  $r^*$  and increasing  $T$ . Therefore, the region of feasible plans is a V-shaped area bounded by LHR and RHR as shown in Figure 1.

In order to search for the plan for the specified prior and specified risk criteria etc., the search region for  $T$  and  $r^*$  is first delineated. We then set  $r^*$  equal to its lower limit and use a binary search method to find a  $T_r$  such that the plan  $(T_r, r^*)$  yields a LHR closest to the desired value, the closeness being specified a-priori. For example, we can require

$$\frac{\text{LHR of } (T_r, r^*) - \text{desired LHR}}{\text{Desired LHR}} \leq .01$$

For these values of  $(T_r, r^*)$ , the corresponding RHR is computed. If the computed value is less than or equal to the desired value then the plan found is the desired one. Otherwise we increase  $r^*$  by 1 unit and continue the search by repeating the above procedure. The search is continued until we find a plan or we do not find one in the delineated region. In the latter case we increase the size of the region and proceed as discussed above. A similar procedure is followed if the RHR is first computed and the test for meeting the LHR criterion is satisfied next. The plans so obtained will not be exactly the same in general but will approximately satisfy the desired risks.

A description of the SUBROUTINE DSP for designing single sample truncated plans using the above procedure is given in Appendix A.

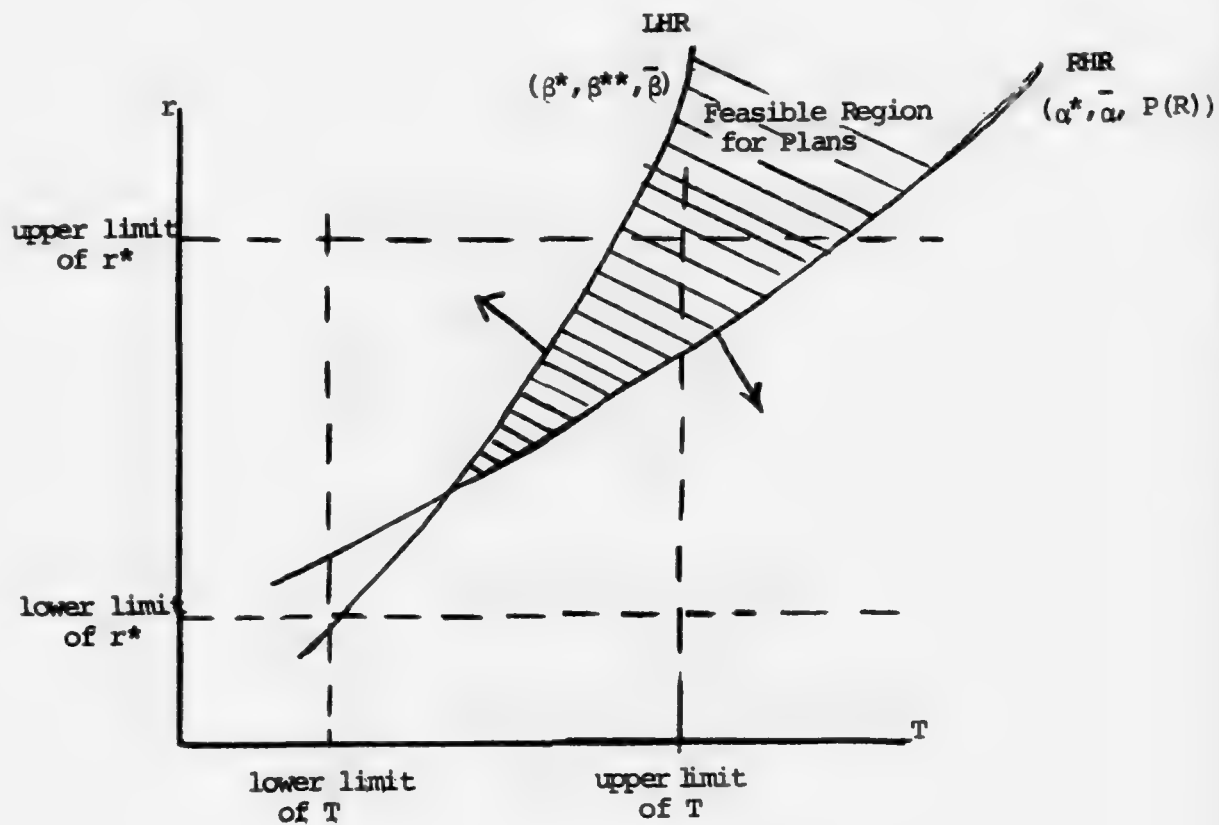


Figure 1. Solution Region For The Numerical Procedure For Plan Design

## 5. GRAPHICAL PROCEDURE FOR THE DESIGN OF PLANS

Given  $(\gamma^*, \lambda, K, \alpha', \beta')$ , where  $\gamma^* = \gamma/\theta_0$ , the graphical procedure permits the computation of  $T^*$  and  $r^*$  where  $\alpha'$  and  $\beta'$  denote the desired producer's and the consumer's risk respectively. The procedure consists of first obtaining contour nomograms by drawing contours of constant  $\alpha'$  and  $\beta'$  in the  $T^* - r^*$  plane. The contours are obtained as follows. In the  $T^* - r^*$  plane a grid is constructed with  $r^* = 0(1)10$  and  $T^* = 0(0.1)9$ . For given  $\lambda$  and  $\gamma^*$  and  $K$ , the values of  $\alpha'$  and  $\beta'$  are evaluated at each of the grid points using SUBROUTINE RISK, given in Appendix B. Finally, contours of constant  $\alpha'$  and  $\beta'$  are drawn by parabolic interpolation. These contours can be used to obtain  $T^*$  and  $r^*$ , for the desired combination of  $\alpha'$  and  $\beta'$ . It should be noted that a new set of contours will be required for each pair of prior parameters  $(\lambda, \gamma^*)$ . However, such contours can be easily obtained by using SUBROUTINE RISK and any subroutine for drawing contours that may be available on a computer system.

As an example, of the design of a test plan using the graphical procedure, consider the design criteria  $\bar{\alpha} = 0.05$  and  $\beta^* = 0.05$ . Let the prior parameters be  $\gamma = 1000$  and  $\lambda = 0.4$ . Let  $\theta_0 = 4000$  and  $\theta_1 = 2000$  such that the discrimination ratio  $K = 2$ . The plots of constant  $\bar{\alpha}$  and  $\beta^*$  are shown in Fig. 2. For  $\bar{\alpha} = 0.05$  and  $\beta^* = 0.05$ , the shaded

zone in Figure 2 represents the solution region, namely, any pair of  $T^*$  and  $r^*$  values in this region implies  $P(A|\theta \geq \theta_0) \leq 0.05$  and  $P(\theta \leq \theta_1|A) \leq 0.05$ . The smallest permissible value of  $r^*$  is 1 and the corresponding value of  $T^*$  is 1. Therefore, the designed plan is  $r^* = 1, T = 4000$ . Various other types of single sample plans can be computed from Table 1 and are given below.

Truncated Plan for a System:  $r^* = 1, T = 4000$

Censored Plan for a System:  $r^* = 2, T = 4000$

Truncated Plan For a Lot With Replacement:  $n = 10, r^* = 1, t_0 = 400$

Censored Plan For a Lot With Replacement:  $r^* = 2, T = 4000$

Truncated Plan For a Lot Without Replacement:  $n = 10, r^* = 1, t_0 = 421$

Censored Plan for a Lot Without Replacement:  $r^* = 2, T = 4000$

The use and advantages of this procedure are now illustrated through several numerical examples.

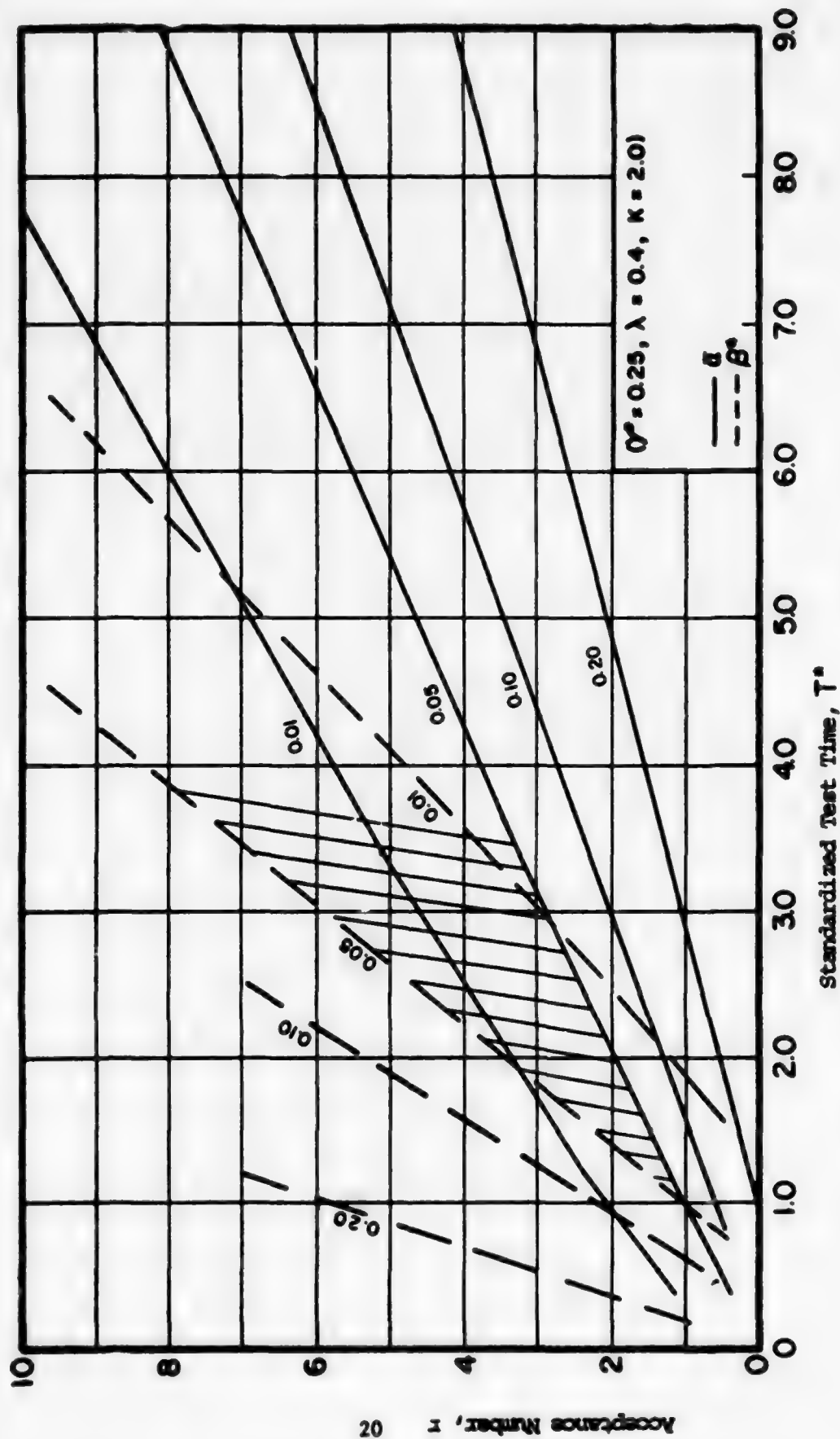


Figure 2. Graphical Design for  $(\bar{\alpha}, \beta^*)$  criteria

### 5.1 Exploration of the Design Region:

An advantage of the graphical procedure is that it permits an exploration of the  $T^* - r^*$  plane near the solution region. Fig. 3 gives the contours of constant  $P(R)$  and  $\beta^*$  for  $\gamma^* = 0.125$ ,  $\lambda = 0.2$  and  $K = 2.0$ . If the design criteria are  $P(R) \leq 0.2$  and  $\beta^* \leq 0.05$ , then the shaded region represents the solution region from which the following table of feasible  $T^*$ ,  $r^*$  values is constructed.

<u><math>r^*</math></u>	<u><math>T^*</math> (<math>\beta^*</math>)</u>	<u><math>T^*</math> (<math>P(R)</math>)</u>
1	-	-
2	1.06	1.15
3	1.42	1.65
4	1.76	2.11
5	2.10	2.60

The notation  $T^*_{(risk)}$  represents the value of  $T^*$  which satisfies the specified risk.

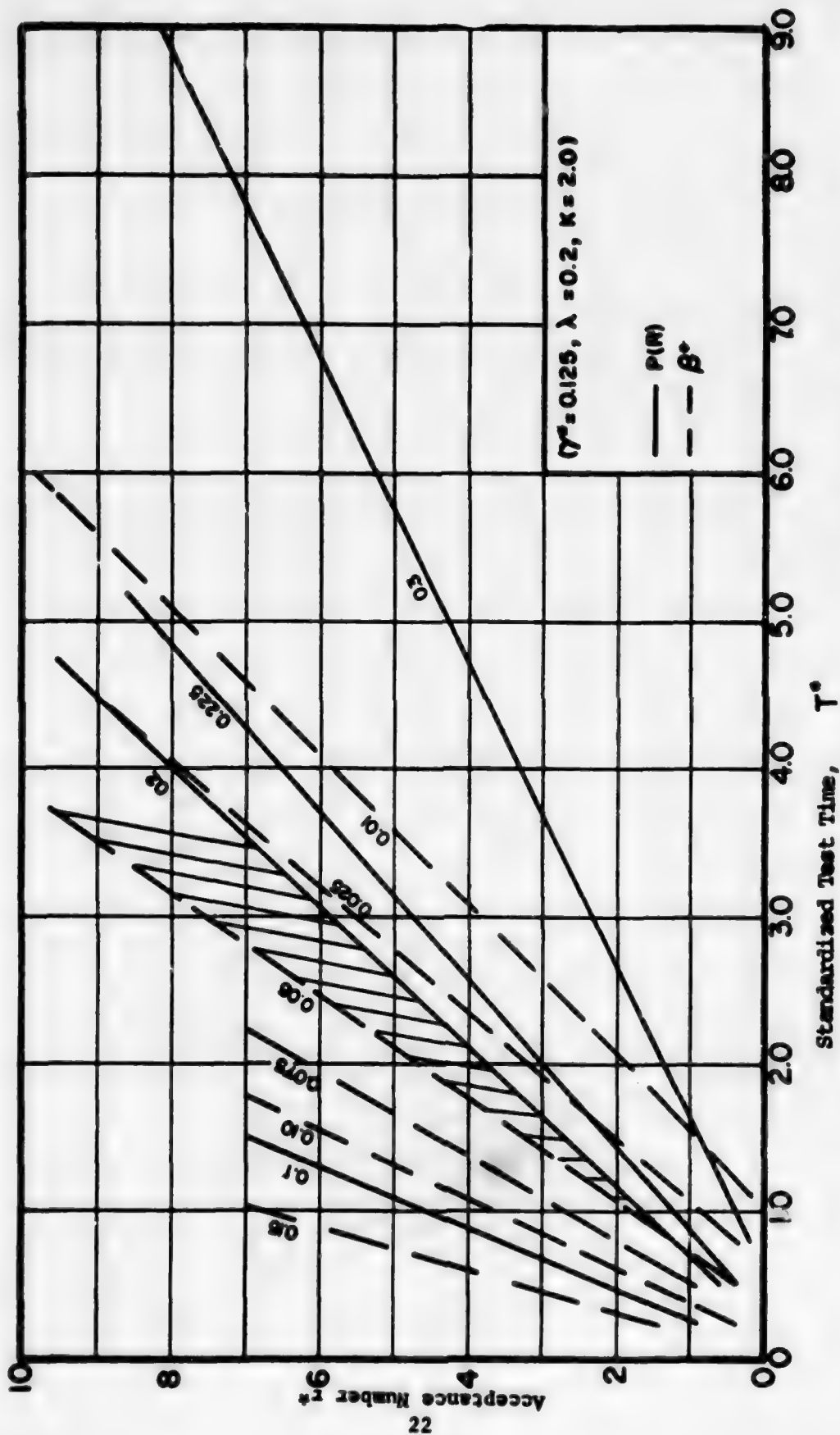


Figure 3. Exploration of the Design Region for  $P(R)$ ,  $\beta^*$  criteria.

Corresponding to a given value of  $r^*$ , the feasible values of  $T^*$  lie between  $T^*_{(\beta^*)}$  and  $T^*_{(P(R))}$ . When  $T^* = T^*_{(\beta^*)}$ ,  $\beta^* = 0.05$  and  $P(R) < 0.2$  while  $T^* = T^*_{(P(R))}$  implies that  $P(R) = 0.2$  and  $\beta^* < 0.05$ . For other feasible values of  $T^*$  in the feasible region, both risks satisfy the inequalities  $P(R) < 0.2$  and  $\beta^* < 0.05$ .

From the viewpoint of testing costs, the solution with minimum  $T^*$  and  $r^*$  is the best. For the present example, such a solution is  $r^* = 2$ ,  $T^* = 1.06$ . Referring to Fig. 3, it is of interest to note that the plan  $r^* = 1$ ,  $T^* = 0.7$  very nearly satisfies the risks with over 33% reduction in test time. This would not be obvious if the plans were designed numerically or presented in a tabular form.

## 5.2 Multiple Risk Criteria:

The graphical procedure is easily extended to the specification of multiple producer-consumer risks. In Fig. 4, the example of Section 5.1 is reconsidered with the added consumer's risk  $\beta^{**}$ . Let the risk criteria be  $P(R) \leq 0.2$ ,  $\beta^* \leq 0.05$  and  $\beta^{**} \leq 0.125$ . These criteria ensure that at most 20% of the systems are rejected, at most 5% of the accepted systems have  $\theta \leq \theta_1$  and at most 12.5% of the accepted systems have  $\theta \leq \theta_0$ .

For these criteria the shaded area in Figure 4 represents the solution region. Compared to the solution region of the previous example, the present solution region is constrained and the dominating criteria are  $P(R)$  and  $\beta^{**}$ . From Figure 4, the designed plan for this case is  $r^* = 3$ ,  $T^* = 1.6$ .

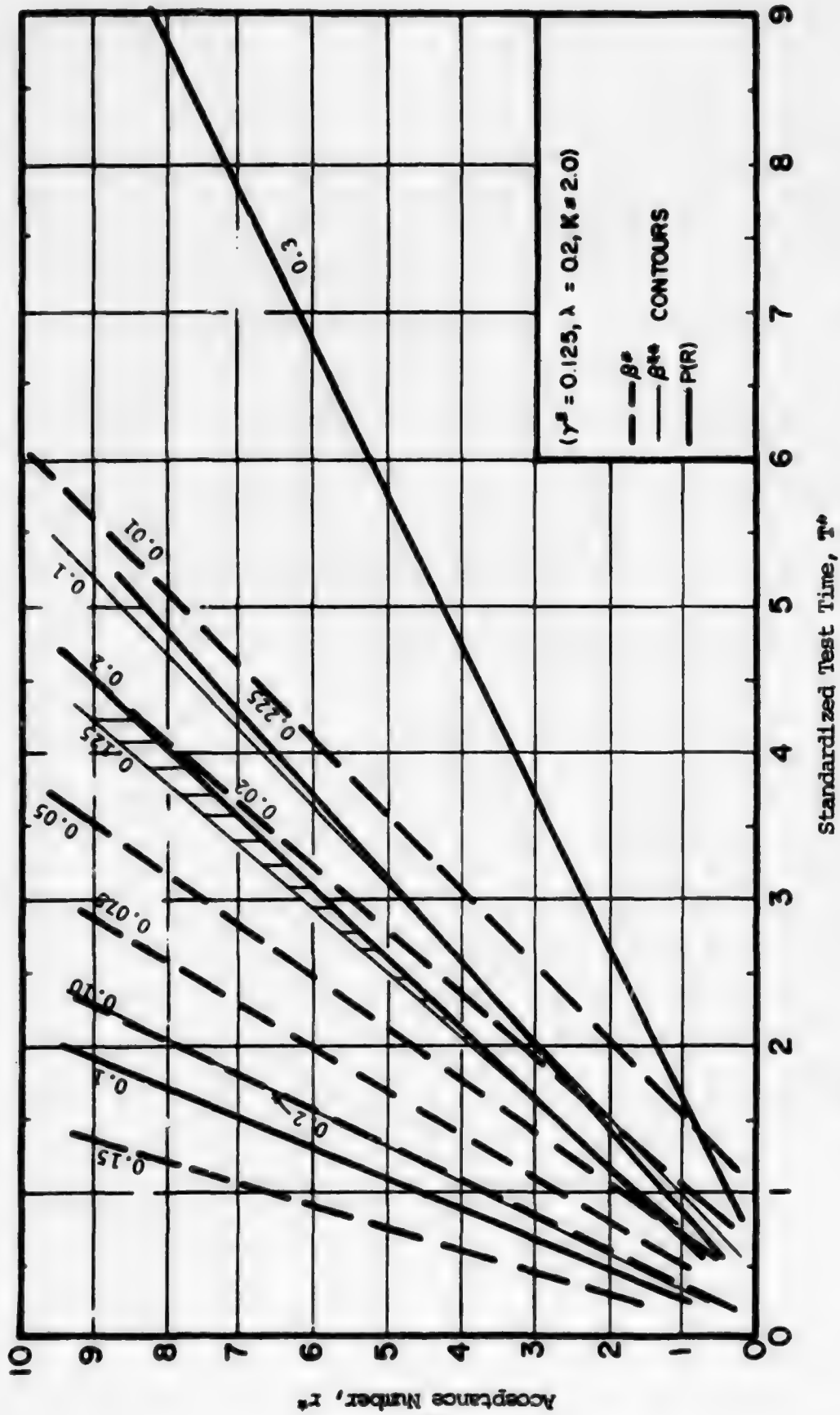


Figure 4. Graphical Design for Multiple Risk criteria,  $(P(R), \beta^*, \beta^{**})$

The graphical procedure also indicates the feasibility of specified risk criteria. For example, if  $\beta^{**} \leq 0.20$ , then this criterion can not be met in practice for the desired values of  $\beta^*$  and  $P(R)$ . If  $\beta^{**} > 0.20$ , however, the governing risks are  $P(R)$  and  $\beta^*$  and the test plan derived in Section 5.1 remains unchanged.

An example of two producer's risks is given in Fig. 5. The criteria are  $P(R) \leq 0.2$ ,  $\bar{\alpha} \leq 0.05$  and  $\beta^* \leq 0.05$ . Solution regions for  $(P(R), \beta^*)$  and for  $(\bar{\alpha}, \beta^*)$  criteria are as shown. Clearly, the  $P(R)$ ,  $\beta^*$  criterion governs and the designed plan is  $r^* = 2$  and  $T^* = 1.25$ .

### 5.3 Design With Engineering Constraints:

At times engineering considerations will require that testing be discontinued after a prefixed time or that only a prespecified number of failures be allowed. The contour nomograms can be used to design an appropriate plan under these constraints. The procedure is illustrated by reconsidering the example of Section 5.1, and referring to Figure 4.

(a) Let us assume that  $T^* \leq 1$ . The following table is constructed from Fig. 4.

<u><math>r^*</math></u>	<u><math>T^*</math></u>	<u><math>P(R)</math></u>	<u><math>\beta^*</math></u>
1	0.5	0.150	0.080
1	0.75	0.205	0.050
1	1.0	0.230	0.030
2	0.5	0.105	0.120
2	0.75	0.140	0.085
2	1.0	0.180	0.060

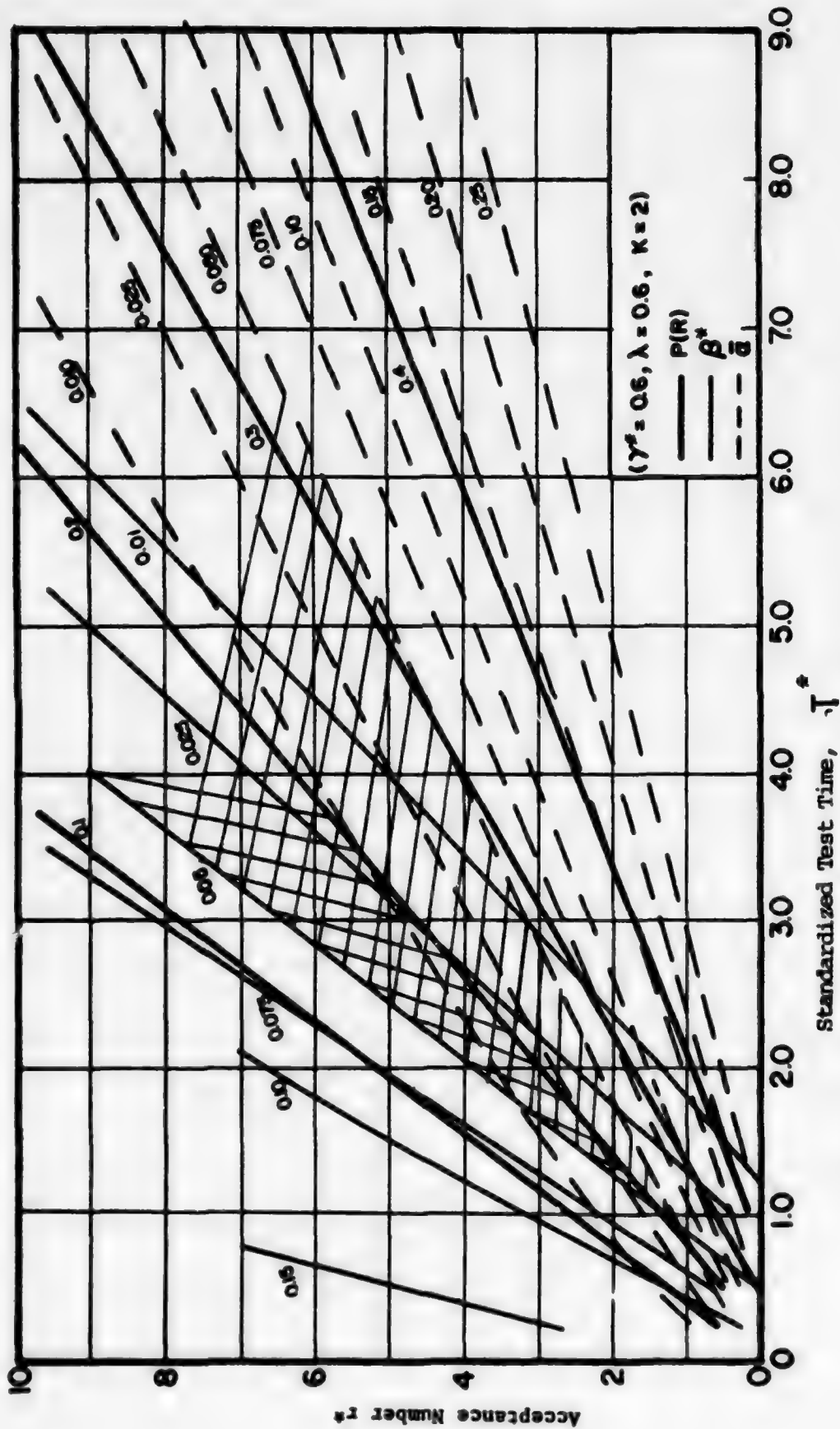


Figure 5. Graphical Design for Multiple Risk Criteria,  $(P(R), \bar{\alpha}, \beta^*)$

A reasonable choice of  $T^*$ ,  $\beta^*$  can now be made. For example, if the desired criteria were  $P(R) \leq 0.2$ ,  $\beta^* \leq 0.05$  then  $r^* = 1$ ,  $T^* = 0.75$  would be the appropriate plan.

(b) Let us now assume that at most two failures are permitted. This implies that we are interested in a censored plan  $(2, T^*)$ . The corresponding truncated plan is  $(1, T^*)$ . The following table is constructed from Fig. 4 to determine an appropriate  $T^*$ .

$r^*$	$T^*$	$P(R)$	$\beta^*$
1	0.5	0.150	0.080
1	0.75	0.205	0.050
1	1.0	0.230	0.030
1	1.25	0.260	0.018
1	1.5	0.285	0.011

Again, if the desired risks were  $P(R) \leq 0.20$  and  $\beta^* \leq 0.05$ , the plan is  $r^* = 1$ ,  $T^* = 0.75$ , the corresponding censored plan being  $r^* = 2$ ,  $T^* = 0.75$ .

#### 5.4 Design With Partial Information:

In some cases only one of the two risks may be specified. For example, a producer may want  $P(R) \leq 0.2$  but the consumer is not certain of the consumer's risk and vice-versa. In such situations a set of alternatives may be put forth as follows.

- (a) Suppose  $P(R) \leq 0.2$  and  $\beta^*$  is unknown. Then the following table can be constructed from Figure 4 by considering the points of intersection of various  $r^*$  values and the  $P(R) = 0.2$  contour.

<u><math>r^*</math></u>	<u><math>T^*</math></u>	<u><math>\beta^*</math></u>
8	4.0	0.026
7	3.53	0.028
6	3.08	0.030
5	2.60	0.032
4	2.10	0.035
3	1.65	0.040
2	1.16	0.044
1	0.70	0.051

From this Table, a suitable plan can be selected.

(b) Suppose  $\beta^* \leq 0.05$  and  $P(R)$  is unspecified. The following table can be constructed from Figure 4 by considering the points of intersection of various  $r^*$  values and the  $\beta^* = 0.05$  contour .

<u><math>r^*</math></u>	<u><math>T^*</math></u>	<u><math>(R)</math></u>
6	2.45	0.166
5	2.10	0.169
4	1.75	0.173
3	1.40	0.181
2	1.07	0.190
1	0.72	0.201

Again, a suitable plan can be selected from this table.

## 6. CONCLUSIONS

1. Equivalence relations have been derived in this report between the single sample truncated, censored, with replacement and without replacement plans. These relations permit the design of all types of single sample plans once the truncated plan for a system is known.
2. A numerical procedure has been presented for the design of single sample truncated plans, when the parameter  $\theta$  of the exponential failure distribution has an inverted gamma prior. This procedure can be used to obtain  $T$  and  $r^*$  for any specified producer's and consumer's risks as given in Equations (14) to (21).
3. A graphical procedure for the design of truncated single sample plan for a system is presented. The procedure is applicable for any specified risk criteria. Other single sample plans can be obtained by using the equivalence relations.
4. Examples are presented to illustrate the advantages of the graphical procedure. Specifically, the examples deal with the design and exploration of the solution region, specification of multiple risk criteria, design under engineering constraints and design with partial information.

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## APPENDIX A

### SUBROUTINE DSP

#### PURPOSE

The purpose of this SUBROUTINE is to search the testing plan  $(T, r^*)$  in a given area for specified parameters of the inverted gamma prior distribution, MTEF  $\theta_0$ , discrimination ratio, and the desired consumer's and producer's risks. The failure time distribution is assumed to be exponential.

The SUBROUTINE also gives the probabilities  $P(\theta \leq \theta_1)$  and  $P(\theta \leq \theta_0)$

#### USAGE

CALL DSP (KL, KR, CX, CY, THO, DR, A, B, TLM, TRM, IR, IRU)

#### DESCRIPTION OF PARAMETERS.

KL	- the consumer's risk	1	for $\beta^*$
		2	for $\bar{\beta}$
		3	for $\beta^{**}$
KR	- the producer's risk	1	for $\alpha^*$
		2	for $\bar{\alpha}$
		3	for $P(A)$
CX	- the specified value of the consumer's risk		
CY	- the specified value of the producer's risk		
THO	- equals $\theta_0$		

- DR - discrimination ratio,  $\theta_0/\theta_1$
- A - scale parameter of the inverted gamma distribution
- B - shape parameter of the inverted gamma distribution
- TLM - the lower limit for searching test time
- TRM - the upper limit for searching test time
- IR - the lower limit for searching acceptance number
- IRU - the upper limit for searching acceptance number

#### SUBROUTINES REQUIRED

GMMA, INVG, MDGAM, UERIST, RISK

#### LISTING

A listing of the SUBROUTINE is given on the following pages.

#### METHOD

The search method for obtaining the test plan  $(T, r^*)$  is described in Section 4 of this report.

```

SUBROUTINE DSP(KL,KR,CX,CY,THO,DR,A,B,TLM,TRM,IR,IRU)
DIMENSION X(3),Y(3)
EQUIVALENCE(X(1),BS),(X(2),QBQ),(X(3),B2S)
EQUIVALENCE(Y(1),AS),(Y(2),QAQ),(Y(3),PR)
TRR=TRM
IF(KR.EQ.3) CY=1.-CY
TH1=THO/DR
ACC=.01
CALL INVG(A,B,THO,PLTHO)
CALL INVG(A,B,TH1,PLTH1)
TYPE 910,PLTH1,PLTHO
910 FORMAT(///' PR. .LE. TH1= ',F7.4,' PR. .LE. THO= ',F7.4)
C**** ACCEPT WITHOUT TESTING
PA=1.
PR=1.-PA
AS=0.
BS=PLTH1
QAQ=0.
QBQ=1.
B2S=PLTHO
IF (X(KL).GE.CX) GO TO 90
IF (Y(KR).GT.CY) GO TO 90
TYPE 930
930 FORMAT(' ***** ACCEPT WITHOUT TESTING ')
TYPE 951,AS,BS,QAQ,QBQ,PA,B2S
RETURN
C90**** REJECT WITHOUT TESTING
90 PA=0.
PR=1.-PA
AS=1.-PLTHO
BS=0.
QAQ=1.
QBQ=0.
B2S=0.
IF (X(KL).GT.CX) GO TO 100
IF (Y(KR).GT.CY) GO TO 100
TYPE 920
920 FORMAT(' **** REJECT WITHOUT TESTING ')
TYPE 951,AS,BS,QAQ,QBQ,PA,B2S
RETURN
C *****STAR SEARCHING SOLUTION
100 TRM=TRR
T=TLM
CALL RISK(T,IR,THO,DR,A,B,AS,BS,QAQ,QBQ,PA,B2S)
PR=1.-PA
IF (CY.GE.Y(KR)) GO TO 12
IF (CX.GE.X(KL)) GO TO 401
IF (IR.GE.IRU) GO TO 250
IR=IR+1
GO TO 100
12 IF (CX.GE.X(KL))GO TO 101
T=TRM
CALL RISK(T,IR,THO,DR,A,B,AS,BS,QAQ,QBQ,PA,B2S)
PR=1.-PA
IF (X(KL).GE.CX) GO TO 200

```

```

      IF (CY.GE.Y(KR)) GO TO 10
      GO TO 40
101  TRM=TLM
      TLM=0.
      GO TO 10
401  TRM=TLM
      TLM=0.
      GO TO 40
40  T=(TLM+TRM)/2.
      CALL RISK (T,IR,THO,DR,A,B,AS,BS,QAQ,QBQ,PA,B2S)
      PR=1.-PA
      IF (CY.GE.Y(KR)) GO TO 20
      IF (CX.GE.X(KL)) GO TO 44
      TLM=T
      IF (IR.GE.IRU) GO TO 250
      IR=IR+1
      GO TO 100
44  TRM=T
      GO TO 40
20  IF (CX.GE.X(KL)) GO TO 102
      TLM=T
      GO TO 40
102 TRM=T
10  T=(TLM+TRM)/2.
      CALL RISK (T,IR,THO,DR,A,B,AS,BS,QAQ,QBQ,PA,B2S)
      PR=1.-PA
      IF (CX.GE.X(KL)) GO TO 111
      TLM=T
      GO TO 10
111 IF ((ABS(CX-X(KL)))/CX .LT. ACC) GO TO 800
      TRM=T
      GO TO 10
250 TYPE 255
255 FORMAT(' **** NO SOLUTION IN THIS AREA, '//
* '      INCREASE NUMBER OF ACCEPTANCE '//
* '      **** THE STOPPING POINT IS')
      TYPE 900,T,IR
      TYPE 951,AS,BS,QAQ,QBQ,PA,B2S
      RETURN
200 TYPE 210
210 FORMAT(' **** NO SOLUTION IN THIS AREA '//
* '      INCREASE TESTING TIME '//
* '      **** THE STOPPING POINT IS')
      TYPE 900,T,IR
      TYPE 951,AS,BS,QAQ,QBQ,PA,B2S
      RETURN
800 TYPE 810
810 FORMAT (' ***** TESTING PLANE IS THE FOLLOWING')
      TYPE 900,T,IR
      TYPE 951,AS,BS,QAQ,QBQ,PA,B2S
      RETURN
951 FORMAT(' A*=',F6.4,2X,'B*=',F6.4,2X,'A-=',F6.4,2X,'B-=',F6.4,2X,
A'PA=',F6.4,3X,'B2S=',F6.4)
900 FORMAT(' T=',F12.3,3X,'R*=',I3,3X,'X=',F8.5,3X,'Y=',F8.5)
      RETURN
      END

```

### Example

Given  $\lambda = 3$ ,  $\gamma = 300$ ,  $\theta_0 = 100$ ,  $DR = 2$ , and let the risk criteria be  $P(A) \leq 0.8$  and  $\beta^* \leq 0.025$ . We search for the testing plan for test time between 0.005 and 150; and for acceptance number between 0 and 3.

### Calling Program and Output

```
CALL DSP(1,3,.025,.8,100.,2.,300.,3.,.005,150.,0,5)
STOP
END
```

```
MAIN.
FORTRAN: DSP
DSP
LOADING
```

```
DDSP ON CORE
EXECUTION
```

```
PR. .LE. TH1= 0.0620 PR. .LE. TH0= 0.4232
*** NO SOLUTION IN THIS AREA
```

```
INCREASE TESTING TIME
```

```
*** THE STOPPING POINT IS
I= 150.000 R*= 3 X=
A*= .1203 B*= .0352 A-= .0209 B-= .5114 PA= .8997 B2S= .3724
STOP
```

Thus we have  $P(\theta \leq \theta_1) = 0.0620$  and  $P(\theta \leq \theta_0) = 0.4232$ . However, the output shows no feasible testing plan in the specified region for  $T$  and  $r$  and suggests increasing the testing time. So we increase the upper limit of test time to 300. The output now is as follows

```
CALL DSP(1,3,.025,.8,100.,2.,300.,3.,150.,300.,0,5)
STOP
END
```

```
exec dsp.rel,ccx.f4,a.rel,rinew.fel
FORTRAN: CCX
MAIN.
LOADING
```

```
CCX 3K CORE
EXECUTION
```

```
PR. .LE. TH1= 0.0620 PR. .LE. TH0= 0.4232
***** TESTING PLANE IS THE FOLLOWING
T= 190.652 R*= 3 X=
A*= .1499 B*= .0248 A-= .0426 B-= .3343 PA= .8332 B2B= .3390
STOP
```

```
END OF EXECUTION
CPU TIME: 0.65 ELAPSED TIME: 4.63
EXIT
```

Thus we get the desired plan to be

$T = 190.654$ , and  $r^* = 3$ .

## APPENDIX B

### SUBROUTINE RISK

#### PURPOSE

This SUBROUTINE computes the risks  $\alpha^*$ ,  $\beta^*$ ,  $\beta^{**}$ ,  $\bar{\alpha}$ ,  $\bar{\beta}$ ,  $P(A)$  for given test time  $T$ , acceptance number  $r^*$ , specified MTBF  $\theta_0$ , discrimination ratio  $K$  and the given parameters of the inverted gamma distribution. The failure time distribution is assumed to be exponential.

#### USAGE

CALL RISK (T,IR,TH0,DR,A,B,AS,BS,QAB,QBQ,PA,R2S)

#### DESCRIPTION OF PARAMETERS

T	- test time
IR	- acceptance number
TH0	- $\theta_0$
DR	- discrimination ratio, equals, $\theta_0/\theta_1$
A	- scale parameter of the inverted gamma distribution
B	- shape parameter of the inverted gamma distribution
AS	- posterior producer's risk, $\alpha^*$
BS	- posterior consumer's risk, $\beta^*$
QAB	- average producer's risk, $\bar{\alpha}$
QBQ	- average consumer's risk, $\bar{\beta}$
PA	- unconditional probability of acceptance, $P(A) = 1 - P(R)$
B2S	- alternate posterior consumer's risk, $\beta^{**}$

#### SUBROUTINES REQUIRED

GMMA, INVG, MDGAM, UERIST

#### LISTING

A listing of the SUBROUTINE is given on the following page.

#### METHOD

The various risks are computed based on the expressions given in Section 3 of this report.

```

SUBROUTINE RISK(T,IR,THO,DR,A,B,AS,BS,QAQ,QBQ,PA,B2S)
DIMENSION PI(99)
IK=IR+1
TH1=THO/DR
CALL INVG(A,B,THO,PLTHO)
CALL INVG(A,B,TH1,PLTH1)
AB=A**B
AT=A+T
ATDTHO=AT/THO
ATDTH1=AT/TH1
ADAT=A/AT
TDAT=T/AT
ADATB=ADAT**B
PI(1)=1.
DO 10 I=2,IK
AI=FLOAT(I-1)
10 PI(I)=PI(I-1)*TDAT*(B+AI-1.)/AI
PAA=0
PB=0.
PBO=0.
DO 20 IIX=1,IK
JJX=IIX-1
TJX=FLOAT(JJX)
W3=TJX+B
CALL MDGAM(ATDTH1,W3,GITH1,IER)
CALL MDGAM(ATDTHO,W3,GITHO,IER)
PAX=PI(IIX)
PJ1=PAX*(1.-GITH1)
PJ0=PAX*(1.-GITHO)
PB=PB+PJ1
PBO=PBO+PJ0
PAA=PAA+PI(IIX)
20 CONTINUE
PB=PB*ADATB
PBO=PBO*ADATB
PA=PAA*ADATB
BS=PB/PA
PR=1.-PA
AS=1.-(PLTHO-PBO)/PR
QAQ=1.-(PA-PBO)/(1.-PLTHO)
QBQ=PB/PLTH1
B2S=PBO/PA
RETURN
END

```

Example:

Given  $\lambda = 3$ ,  $\gamma = 300$ ,  $\theta_0 = 100$ ,  $DR = 2$  and the testing plan,  $T = 190.654$ ,  $r^* = 3$ , we want to obtain the various producer's and the consumer's risks.

Calling Program and Output

```
CALL RISK(190.654,3,100.,2.,300.,3.,AS,BS,QAQ,QBQ,PA,B2S)
TYPE 10,AS,BS,QAQ,QBQ,PA,B2S
10  FORMAT(' ',6F10.4)
STOP
END
```

```
CCX 2K CORE
EXECUTION
    0.1499    0.0248    0.0426    0.3343    0.8362    0.3396
STOP

END OF EXECUTION
CPU TIME: 0.08  ELAPSED TIME: 3.73
EXIT
```

Thus, for the specified prior distribution,  $\theta_0$ ,  $DR$  and the test plan, the various risks as computed above are:

$$\alpha^* = 0.1499, \beta^* = 0.0248, \bar{\alpha} = 0.0426, \bar{\beta} = 0.3343, P(A) = 0.8362, \\ \beta^{**} = 0.3396.$$

APPENDIX C  
SUBROUTINE GAMMA

PURPOSE

This SUBROUTINE computes the gamma function  $\Gamma(x)$  for a given argument  $x$  where

$$\Gamma(x) = \int_0^{\infty} t^{x-1} \cdot e^{-t} dt$$

USAGE

CALL GAMMA (XX,GX,IER)

DESCRIPTION OF PARAMETERS

XX - the argument for the gamma function  
GX - the resultant gamma function value  
IER - resultant error code where

IER = 0    no error  
IER = 1    XX is within 0.000001 of being a negative integer  
IER = 2    XX is greater than 57, overflow, GX set to 1.0E75

SUBROUTINES REQUIRED

NONE

METHOD

The recursion relation and polynomial approximation by C. Hastings, Jr., "Approximations for digital computers," Princeton University Press, 1955.

LISTING

A listing of the SUBROUTINE is given on the following page.

This SUBROUTINE is taken from the International Mathematical and Statistical Libraries.

```

SUBROUTINE GMMMA(XX,GX,IER)
  IF(XX-57.)6,6,4
4  IER=2
  GX=1.E75
  RETURN
6  X=XX
  ERR=1.0E-6
  IER=0
  GX=1.0
  IF(X-2.0)50,50,15
10 IF(X-2.0)110,110,15
15 X=X-1.0
  GX=GX*X
  GO TO 10
50 IF(X-1.0)60,120,110
C
C      SEE IF X IS NEAR NEGATIVE INTEGER OR ZERO
C
60 IF(X-ERR)62,62,80
62 Y=FLOAT(INT(X))-X
  IF(ABS(Y)-ERR)130,130,64
64 IF(1.0-Y-ERR)130,130,70
C
C      X NOT NEAR A NEGATIVE INTEGER OR ZERO
C
70 IF(X-1.0)80,80,110
80 GX=GX/X
  X=X+1.0
  GO TO 70
110 Y=X-1.0
  GY=1.0+Y*(-0.5771017+Y*(+0.9858540+Y*(-1.3754218+YY(+0.11212
  1Y*(-0.5684729+Y*(+0.2548205+Y*(-0.05149930))))))
  GX=GX*GY
120 RETURN
130 IER=1
  RETURN
  END

```

## APPENDIX D

### SUBROUTINE INVG

#### PURPOSE

This SUBROUTINE computes the cumulative distribution function of the inverted gamma density  $f(\theta; A, B)$  for a given argument  $\theta$  with scale parameter  $A$ , and shape parameter  $B$ .

#### USAGE

CALL INVG (A,B,TH,P)

#### DESCRIPTION OF PARAMETERS

A - scale parameter of the inverted gamma distribution  
B - shape parameter of the inverted gamma distribution  
TH -  $\theta$   
P -  $P(\theta \leq \theta)$

#### SUBROUTINES REQUIRED

MDGAM

#### LISTING

This SUBROUTINE is taken from the International Mathematical and Statistical Libraries.

```
SUBROUTINE INVG(A,B,TH,P)
X=A/TH
CALL MDGAM(X,B,P,IER)
P=1.-P
RETURN
END
```

APPENDIX E  
SUBROUTINE MDGAM

PURPOSE

MDGAM evaluates the probability that a random variable from a gamma distribution having the parameter P is less than or equal to X. Thus MDGAM computes the incomplete gamma ratio

$$I(X,P) = \int_0^X e^{-t} t^{P-1} dt / \Gamma(P)$$

where  $\Gamma(P)$  is the complete gamma function.

USAGE

CALL MDGAM (X,P,PROB,IER)

PARAMETERS

- X - value to which gamma is to be integrated
  - P - input gamma parameter
  - PROB - output probability = integral of gamma (P) to X
  - IER - error indicator
- Terminal error = 128 + N
- N = 1 indicates X is less than zero
- N = 2 indicates P is less than or equal to zero

LISTING

The listing is given on the following page.

This SUBROUTINE is taken from the International Mathematical and Statistical Libraries.

```

SUBROUTINE MDGAM (X,P,PROB,IER)
C
  DIMENSION          V(6),V1(6)
  EQUIVALENCE        (V(3),V1(1))
C                                TEST X AND P
  PROB = 0.0
  IF (X .GE. 0.0) GO TO 5
  IER = 129
  GO TO 9000
5 IF (P .GT. 0.0) GO TO 10
  IER = 130
  GO TO 9000
10 IER = 0
  IF (X .EQ. 0.0) GO TO 9005
C                                DEFINE LOG-GAMMA AND INITIALIZE
  PNLG = ALGAMA(P)
  CNT = P * ALOG(X)
  YCNT = X + PNLG
  IF ((CNT-YCNT) .GT. -174.673) GO TO 15
  AX = 0.0
  GO TO 20
15 AX = EXP(CNT-YCNT)
20 BIG = 1.E35
  CUT = 1.E-8
C                                CHOOSE ALGORITHMIC METHOD
C                                CONTINUED FRACTION EXPANSION
  IF ((X .LE. 1.0) .OR. (X .LT. P )) GO TO 40
  Y = 1.0 - P
  Z = X + Y + 1.0
  CNT = 0.0
  V(1) = 1.0
  V(2) = X
  V(3) = X + 1.0
  V(4) = Z * X
  PROB = V(3)/V(4)
25 CNT = CNT + 1.0
  Y = Y + 1.0
  Z = Z + 2.0
  YCNT = Y * CNT
  V(5) = V1(1) * Z - V(1) * YCNT
  V(6) = V1(2) * Z - V(2) * YCNT
  IF (V(6) .EQ. 0.0) GO TO 50
  RATIO = V(5)/V(6)
  REDUC = ABS(PROB-RATIO)
  IF (REDUC .GT. CUT) GO TO 30
  IF (REDUC .LE. RATIO*CUT) GO TO 35
30 PROB = RATIO

```

```

      GO TO 50
35  PROB = 1.0 - PROB * AX
      GO TO 9005
C
      SERIES EXPANSION
40  RATIO = P
      CNT = 1.0
      PROB = 1.0
45  RATIO = RATIO + 1.0
      CNT = CNT * X/RATIO
      PROB = PROB + CNT
      IF (CNT .GT. CUT) GO TO 45
      PROB = PROB * AX/P
      GO TO 9005
50  DO 55 I=1,4
      V(I) = V1(I)
55  CONTINUE
      IF (ABS(V(5)) .LT. BIG) GO TO 25
C
      SCALE TERMS DOWN TO PREVENT OVERFLOW
      DO 60 I=1,4
      V(I) = V(I)/BIG
60  CONTINUE
      GO TO 25
9000 CONTINUE
      CALL UERTST (IER,6HMDGAM )
9005 RETURN
      END
      FUNCTION ALGAMA(P)
      CALL GMMMA(P,GP,IER)
      ALGAMA=ALOG(GP)
      RETURN
      END

```

APPENDIX F  
SUBROUTINE UERTST

PURPOSE

To print a message reflecting the error detected by a SUBROUTINE

USAGE

CALL UERTST (IER,NAME)

PARAMETERS

IER     -   error parameter. Type + N where

          type = 128 implies terminal error

          64 implies warning with fix

          32 implies warning

          N     = error code relevant to calling routine

NAME     -   input vector containing the name of the calling routine  
          as a six character literal string.

LISTING

Given on the following page.

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```

SUBROUTINE UERTST(IER,NAME)
  DIMENSION      ITYP(5,4),IBIT(4)
  INTEGER        NAME(6)
  INTEGER        WARN,WARF,TERM,PRINTR
  EQUIVALENCE    (ITYP(1),WARN),(IBIT(2),WARF),(IBIT(3),TERM)
  DATA ((ITYP(I,J),I=1,5),J=1,4)
  *              /'WARN','INC',' ',' ',' ',' '
  *              /'WARN','INC','WITH','FIX',' ',' '
  *              /'TERM','INAL',' ',' ',' ',' '
  *              /'NON-','DEFI','NE',' ',' ',' '
  DATA (IBIT(I),I=1,4)/ 32,64,128,0/
  DATA      PRINTR      / 6/
  IER2=IER
  IF (IER2 .GE. WARN) GO TO 5
C                                     NON-DEFINED
    IER1=4
    GO TO 20
  5 IF (IER2 .LT. TERM) GO TO 10
C                                     TERMINAL
    IER1=3
    GO TO 20
  10 IF (IER2 .LT. WARF) GO TO 15
C                                     WARNING(WITH FIX)
    IER1=2
    GO TO 20
C                                     WARNING
  15 IER1=1
C                                     EXTRACT 'N'
  20 IER2=IER2-IBIT(IER1)
C                                     PRINT ERROR MESSAGE
    WRITE (PRINTR,25) (ITYP(I,IER1),I=1,5),NAME,IER2,IER
  25 FORMAT(' *** I M S L(UERTST) *** ',5A1,4X,3A2,4X,12)
  *      (IER = ',13,')
  RETURN
  END

```